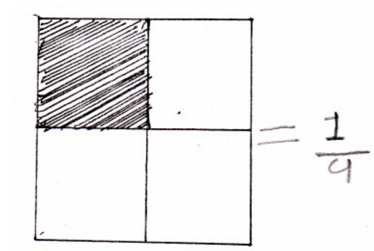
Teaching fractions - Brajesh[1]

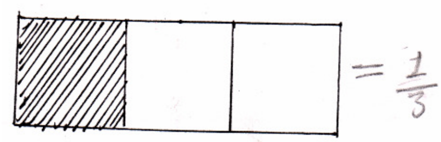
The term 'fraction' in its literal sense reflects a part of a whole. That is how we've always learnt fractions in school and also as a teacher, transferred the concept to children. Within our centre, we thought of several ways of making it easier for children to understand it. We procured and developed teaching-learning aids where we could show ¼, ½, ¾, ⅓, ⅔, etc. There was an interesting variety of aids put together. However I did not feel very sure of it. While there was merit in what I was doing in class, there were some areas where I felt I was digressing from the principle of learning through practical experience and was going into area of algorithmic expectations .

Therefore when we were approached by team members of Eklavya and Homi Bhabha Science Centre to try out the model of conveying fractions with our group of children through the 'sharing' model, I was keen to try it out in my class.

Difficulties with using only the conventional measure model in class

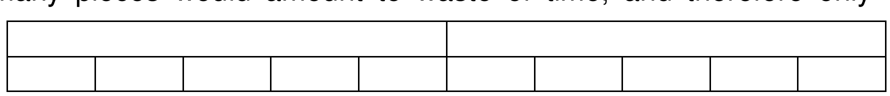
The measure model, by its definition, implies that the number is supposed to reflect a quantity; it is a measure of a specific quantity. It is not so challenging to teach unit fractions, as one–third, one-half, one-fourth because it can be introduced to the children as a number which is less than the whole number and therefore the need to find a way of writing something less than one.





When the fraction is a composite fraction i.e. conveys a value over 1, as 5/4, then there is another step that goes in to explain that we would draw out one more of the ¼ size. In usual circumstances, it does not happen that we approach a shop and say that we need $^5/_4$ of a meter of cloth; it is usually said as 1 and ¼ and nor does one buy 5 packets of ¼ kg. of potatoes often.
It is not for exceptional days or for a later evolved form of fractions that we introduce fractions in classes 4^{th} and 5^{th}. There has to be logic to why we are learning something in its present form also which has to be conveyed to the children indirectly in its method of teaching, and should not be expected that its functional value is 'in the future'.

It becomes further complicated when we suggest that to take ½ of something, it is equivalent to dividing the whole thing in 20 parts and then taking 10, to point out that $^{10}/_{20}$ is equivalent to ½. In reality, this is again something which is not done. The distribution of something in so many pieces would amount to waste of time, and therefore only serves the purpose of writing it in one's notebooks.



[1] Brijesh works in Muskaan, an NGO working amongst slum communities in Bhopal. These are his experiences in the learning centre for the children.

The problem here is not that the children find it difficult to understand because it can be shown and seen; the problem is of practical usage.

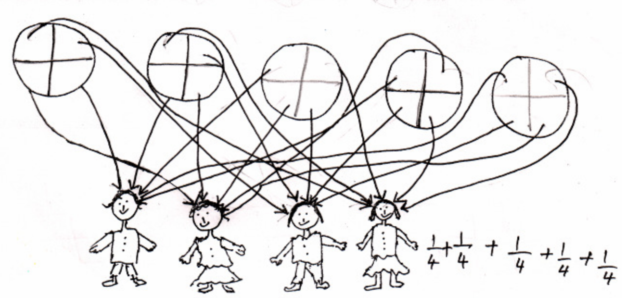
The problem takes another dimension when any operations have to be done on the fractions (as addition, subtraction) or when two fractions have to be compared. The only ways we would resort to were completely algorithmic, get a common denominator because then only comparisons can be made or things can be added.

When algorithms are introduced in a preliminary stage of learning, there is a real threat of the basic number value getting lost in the emphasis of the method. Therefore we find that many children and adults fear fractions. Our experience has been that majority of adults, when given a problem of fractions, as ½ + $^{3}/_{2}$, their response is either that they are not comfortable with mathematics or they would initiate a long-drawn method of solving this sum, wherein there are more chances of their getting it wrong than there would be of their getting it right.

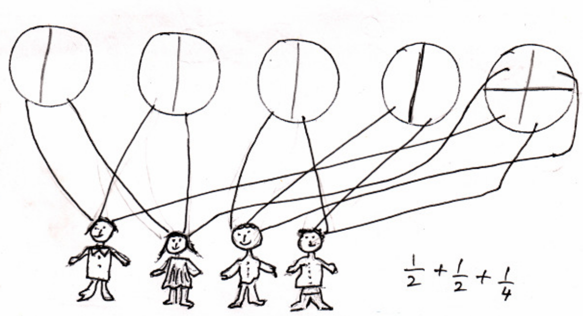
Teaching fractions through the 'sharing' model

Unit fractions or composite fractions - Children have enjoyed doing fractions through the sharing model because there they get to divide the things in whatever way they want to. Children were initially told that there are 5 chapattis and they need to be distributed amongst 4 children i.e. $^{5}/_{4}$, as we would write 5 divided by 4 and would it be possible that everyone gets an equal share.

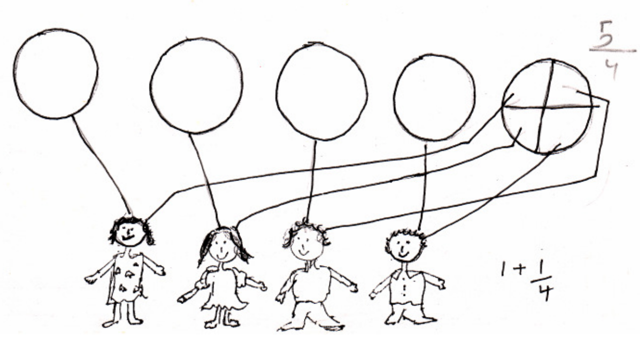
A child found three ways to do this.



Here, the child has divided the first chapatti in four parts and given one part to each, and then the parts of the second chapatti have been distributed to each and then the last chapatti till the end.
¼ + ¼ + ¼ + ¼ + ¼



Here, the child has divided the first chapatti in two parts and then the next three also in two parts, and given each of the four children two half parts each. Then he has divided the last chapatti in four parts and given a part each.
½ + ½ + ¼



Here, the child has given four chapattis to four children each. Of the last chapatti, he has made 4 parts and given each part equally from this last chapatti to each child.
1 + ¼

I realized that as children found ways to explore one concept, they had gone into other dimensions of fractions (as addition of fractions in the above example), and therefore when we reach the second stage of fractions, we would not have to begin afresh or in a manner in which I would need to explain or that it would become a new unit to be taught. It simply grew to be an inference from what they were already doing and discovering on their own.

Equivalent fractions – Extending the example of distribution of chapattis amongst the children, it was discussed that
- If we distributed one roti between two children, i.e. $\frac{1}{2}$, how much would each child get?
- If we distributed 5 chapattis in 10 children, i.e. $\frac{5}{10}$, then how much would each get?

Children started enjoying this and made umpteen continuous fractions, going into thousands and lakhs.

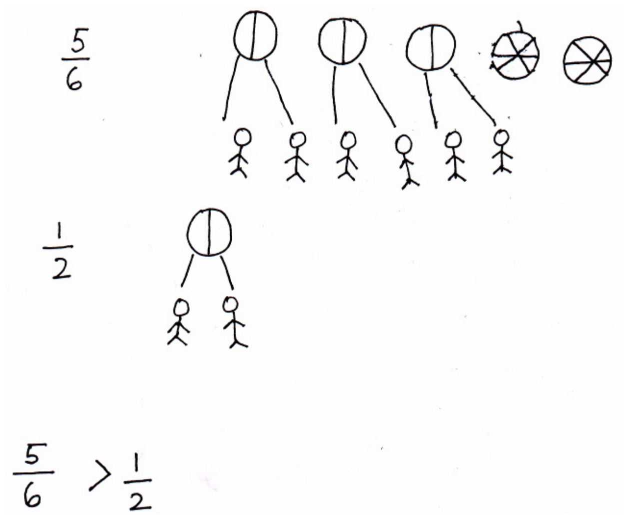
Comparison – When we delved into comparison of fractions, I realized that children were building their own systems of reaching an answer. I could offer clues, but it may be taken by one child and not by another. Sometimes, a child who had reached a conclusion through one method would convince the others on that answer (but not the method).

When I put forth the question as which of the two fractions is bigger - $\frac{3}{7}$ and $\frac{5}{9}$, one child responded that when I distribute 5 chapattis in 9 persons, then I have a balance of $\frac{1}{2}$ chapatti left after giving each one $\frac{1}{2}$ a chapatti, but when I distribute 3 chapattis in 7 persons, then if I have to give one $\frac{1}{2}$ to each, then one person gets left out so I will have to take back from all the others to make an equal distribution. So $\frac{5}{9}$ is bigger because everyone is getting more than $\frac{1}{2}$ there, and in $\frac{3}{7}$, everyone would get less than $\frac{1}{2}$.

On another question, which is bigger between $\frac{8}{12}$ and $\frac{9}{8}$, one child responded that compared to the first fraction, the number of chapattis have been increased in the second fraction (the numerator has gone up from 8 to 9), and the number of people in whom the food has to be distributed has also decreased (the denominator has reduced from 12 to 8). Therefore $\frac{9}{8}$ is more. Another child explained that he found $\frac{9}{8}$ bigger because there were more chapattis and less people, but in the $\frac{8}{12}$ fraction, there were less chapattis and more people.

When the problem as to put the fractions in an ascending order $\frac{4}{5}$, $\frac{4}{8}$, $\frac{4}{12}$, $\frac{4}{9}$ was put to the children, they got it right immediately. On how did they do it, most of the children answered that there is a constant of 4 chapattis and when the number of people within whom it is to be distributed decreases, each person will get a larger share.
There was one other child whose logic worked differently. He explained that:
- in $\frac{4}{12}$, each person got a share of $\frac{1}{3}$;
- in $\frac{4}{9}$, 1 whole chapatti was left to be distributed in 9 persons after giving each person $\frac{1}{3}$, so each would get $\frac{1}{3}$ and $\frac{1}{9}$
- in $\frac{4}{8}$, each person was getting $\frac{1}{2}$, and
- in $\frac{4}{5}$, each person got $\frac{1}{3}$ chapatti and then there was still 1 whole chapatti and $\frac{1}{3}$ left for distribution in 5 persons.

- Therefore $\frac{4}{5}$ is the biggest. Surprisingly, the child also visualized accurately between $\frac{4}{9}$ and $\frac{4}{8}$ to suggest that ½ would be more than ⅓ + $\frac{1}{9}$ (the child was sure this is less than half, using the other logic that there is one more person to whom the 4 chapattis have to be distributed). And $\frac{4}{12}$ is the smallest.



Here, the child has distributed the first three chapattis in two halves and realized that each person will get more than a half because she had two complete chapattis left after distributing the first three. In comparison, in the case of ½ , she cannot give any more to each person. So the first is visibly more than the second.

Addition – When any two fractions had to be added, then the method had been to present the fractions per se, and depict it on a shape, and overlap the two shapes over each other. This effort somewhere tends to underplay the measure in itself, as it does not denote a quantity (in numeric value), but becomes two shapes to be merged.

When ½ and $\frac{1}{3}$ have to be added, then the purpose of the question should be how much does each get when two quantities are being merged. Simply, if I get 2 toffees and 3 toffees, how many do I have = 5 toffees. But when we apply this in fractions, the complication of not being able to visualize it nor relate to it, then the complexity of the situation grows.
If presented as mother was making chapattis and there were 6 children waiting, so when she had made 3 chapattis, she distributed it equally amongst them, and then she got 2 more chapattis and did the same. How much each child gets, and the children were able to respond that first they got ½ each and then $\frac{1}{3}$ each, and there were 5 chapattis distributed in 6 persons, so $\frac{5}{6}$.

The children had to be reminded to use the concept of equivalent fractions during addition with the logic that there had to be a common denominator reflecting the constant number of persons where things were being distributed. Based on this clue, the children were able to do further problems of additions.

While initially the children were doing it sequentially, as
1/4 = 2/8 = 3/12 = 4/16 = 5/20
2/5 = 4/10 = 6/15 = 8/20

But soon enough, several children found short-cuts to save time and energy, and said that if the numerator is being increased 5 times, then so is the denominator, and to find a common denominator, we have to multiply 4 and 5. (This may not be the lowest multiple in all the cases, and when children have enough of these kinds of problems to solve, they are very capable of reaching that logic too).

Supplementing – both models

At different points, we can see that it is not simply a choice of one model over the other. The two start flowing into each other, and it becomes easier for the child to see it. When one chapatti is distributed in three persons, it is conveyed as 1 chapatti / 3 persons, and what each person gets is also $1/3^{rd}$ of the total available.
The sharing model conveys the meaning of the measure in further depth, and therefore does not need to be seen separately. The measure form of the fraction was specifically introduced to the children during this process.

We can note this in the comparisons of fractions also where the fraction reflects a measure and therefore it is not an action of dividing 3 chapattis in 7 people, but is the value of the share of each person on this distribution, but the answer as to which is bigger adopts a process of looking at it from a sharing perspective and then comparing it from a measure perspective.

The concept of fractions as a measure and a proportion was extended to different materials, as money, quantities and rates of things of daily use (as sugar, rice, etc.).

Drawings from the class in fractions -

- Examples used at every stage of introducing the concepts were taken from the local environment. The children that we were teaching lived in slums and their life produced situations very different from what is depicted in the regular books. Therefore, we were careful that instead of the usual pizzas and cakes to be distributed, it was chapattis or a set of pencils. Therefore the available books (even published in India) could not be directly used.
  It was not only about contextualizing the thing to be distributed, the value also needed to be considered. Thus when there was a discussion on how much amount would come on each person if they took ½ or ¼ of a packet of a washing powder and the children were surprised that they had to distribute the packet which costs Rs. 30 in the market; the first response was that the (whole) packet should be of Rs. 3 or Rs. 10 because they would never spend so much money on washing powder at a time.

- In our efforts in class, there is always a need to look for relevant teaching-learning materials to explain a concept. But after taking this class in fractions, we felt a need to re-look on the emphasis of TLM. Activities and TLM need to be created outside a classroom if the concept is alienated from the child's life. However, this should always (and not sometimes) be a two-pronged process where the in-class discussions and children's lives are used to make the TLM or guide an activity.
  Most of the content in elementary classes is around consolidating the information children already knows into a written form, and thus finding ways to write and depict on paper concepts emergent from life, but the method of teaching distances the concept from reality. In teaching fractions, our perception of fraction has also become a circle and division of a circle, etc. but it could be seen as a contract

amount distributed in a group of labourers, which is a distribution as well as a sharing model.

- When giving children a test, we need to be sure on what we want to test the child at. Many a times, children falter on the language of the question, and on other times, they may know the concept verbally but don't know how to write it. However if the emphasis is on the written form, it should be ensured that at least the former problem does not take place. Complicated questions and language also deter children and put them off, and many of them do not get into it.
  Since we were taking inputs from external individuals who were specialists in mathematics, we would often feel the need to simplify the way a desired action was put forth to the children. For instance, the same question of distribution of a certain amount to different people and asking how much each got in one sentence became complicated for some children, but if it was put in one question after the other question, the children managed it. It is natural to move to further degrees of complexity within academics, but the time for taking this step could be different for different children. If the complexity is only for the adult's convenience and does not really affect the understanding of the concept per se, then it should be done away with.

- There should be an effort to keep the intrinsic value of a natural number or a fraction in perspective. Usually, a fraction could be seen in reference to whether it is less/more than half or less/more than 1. Through the method we had adopted, we found that the children were able to estimate well. Children immediately responded to whether someone is getting more than one, or about one-half and therefore will it be more or less than the other. In conversation, a child would say 'just a little bit more to convey that he was not substantially changing the number'.

- Usually, we try and use earlier mathematical concepts to explain the new one, which is fair and not questionable per se. In elementary classes, it is better to be able to make logic, not only in the abstract sense (i.e. numerical), but in practical senses also. For example, when discussing equivalent fractions, the tendency is that if you are multiplying the numerator by a certain number, then the denominator should also be multiplied by that number. Children would reach the logic on their own if there is enough time and mental space to work it out. Thus when if we start from the point of how much should be increased in the numerator or the denominator if either one is changed to a certain value so that the result is same, majority of the children would reach the right conclusion.
  Activities thus should be designed to provide opportunities for children to work out instead of giving a method. Method can be developed later and children also start developing the method on their own; this does not happen as a product of deliberate teaching but as an evolved written sequence of thought in the shape of an algorithm at a later stage.

- In most 'child-friendly' spaces including ours, we think that there is a space for children to do things as per their mind. But when the teacher is not confident of the concept oneself, then it becomes challenging for one to allow the children to mangle it any which way becauset hen one is not able to always respond to a child's queires. Teahcers then insist on why they believe a certain thing should be done in a certain way, and don't have to be bound to give answers to children's questions if the child tried it any other way. It is one thing to provide the space in the teaching of a language subject where we express our thoughts, but also in mathematics. How would one divide 332 in 2 persons, someone could be comfortable in dividing 100 and 100 first, and then think of what to do with the left 132; another could do it 150 and 150, and then find different ways to distribute 32, etc. In the fractions class, children gave their answers to questions posed and then explained how they had reached that answer. If the answer was wrong, it needed to be seen where the child' logic had gone wrong mathematically, and not in changing the method completely.

- In the above situation, when children feel in control of the classroom process, comfortable with the space, teacher, and the subject, then they start experimenting more with the subject also and do not avoid any concept which may be typically considered 'difficult'.
  We also often hear that working children find it difficult to go to school or study or remember things they have been taught earlier. But such methods of teaching showed that children who were considered 'difficult' to come to the centre to study because they had to work or did not enjoy class within their independent spirit, were also coming regularly. It is in the methods of teaching and classroom processes to determine the response of the children on a subject, teachers and schooling per se.

A film has been made on the classroom processes in teaching fractions through this method and depicts the children's responses naturally. A copy could be made available on sending a request to muskaan_smiles@hotmail.com